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Models of Educational Stages

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Models of Educational Stages

Abstract

Invited by the National Educational Panel Study (NEPS), during the Winter 2011/12, I gave a series of lectures about 'Statistical methods in sociological research of education'. This text comprises an elaboration of two of the lectures discussing models of educational stages. I consider two kinds of such models, both dealing with a sequence of two learning frames. Models of the first kind assume that individuals sequentially participate in both learning frames; models of the second kind involve a selection process that concerns transitions to the second learning frame. Following the introduction of these two kinds of models, I take up the distinction between primary and secondary effects of the family background. I first consider the standard approach where primary effects concern individual's level of academic performance, and secondary effects concern transition probabilities given fixed performance levels. I then introduce an alternative approach where primary effects concern individual's opportunities to make a transition (which in turn may depend on academic performance), and secondary effects correspondingly concern how they actually use these opportunities.

Keywords

educational stages, educational inequality, primary and secondary effects, functional models

Invited by the National Educational Panel Study (NEPS), during the Winter 2011/12, I gave a series of lectures about 'Statistical methods in sociological research of education'. This text comprises an elaboration of two of the lectures discussing models of educational stages.

As a starting point, I take up the idea that education takes place in a wide variety of 'learning environments' (Bäumer et al. 2011). Formal representations of such environments will be called *learning frames* and symbolically denoted by σ. There definition should include: a characterization of the institutional (not necessarily 'formal') context; a characterization of the kind of capabilities that can be learned; a characterization of the ways by which these capabilities can be acquired; specification of entry requirements (if any); definition of an outcome space, say $\mathcal{Y}_\sigma$, for defining variables representing what has been learned. The general notion of learning frames covers both broad (e.g. 'elementary school') and fine-grained (e.g. 'learning reading in first grade elementary school') definitions. This must be taken into account when defining the outcome space $\mathcal{Y}_\sigma$. In order to represent outcomes of broadly defined learning frames it could be sensible to use outcome spaces that consist of two or more domain-specific components.

A further question concerns the source of 'observed' outcome values. There are two approaches: One can use the evaluations that are generated in the learning frames (e.g. school certificates), or one can employ external tests developed and applied by a researcher. Choosing one or the other approach depends on the research interest. The first approach corresponds to an interest in learning how educational outcomes are actually generated and evaluated in a society. The second approach corresponds to an interest in developing scientific evaluations. In the present text, I am neutral on this point and assume that outcome values are given in some way. This is possible because I only use fictitious data for illustrations.

As a formal framework, I use functional models (see Rohwer (2010, 2012) for notations and definitions). In this text, they represent educational processes which consist of two consecutive learning frames, σ_1 and σ_2. The first section considers models of consecutive educational outcomes which presuppose that individuals participate in both learning frames. The second section considers models of sequential transitions which entail a selection process. The third section discusses distinctions between primary and secondary effects of the family background. The fourth section considers an enlarged model in which transitions depend on previous educational outcomes.

1. Consecutive educational outcomes

1. Framework of a two-stage model. I consider a sequence of two learning frames, σ_1 and σ_2. Let $\dot{Y}_t$ denote the outcome of learning frame σ_t $(t = 1, 2)$, based on some measure of what has been learned. I assume that these are binary or quantitative variables so that one can sensibly refer to expectations. I consider a model which posits that these outcome variables depend on two explanatory variables, say $\ddot{X}$ (e.g. a measure of socio-economic status) and $\ddot{Z}$ (e.g. an indicator of educational aspiration). The model can be depicted as follows:

Model 1

$$\begin{tikzcd}
\ddot{X} \arrow[d] \arrow[dr] & \\
\dot{Y}_1 \arrow[r] & \dot{Y}_2 \\
\ddot{Z} \arrow[u] \arrow[ur] &
\end{tikzcd}$$

Table 1 Fictitious data for the two-stage model.

x	z	y_1	y_2	cases	x	z	y_1	y_2	cases
0	0	0	0	300	1	0	0	0	200
0	0	0	1	400	1	0	0	1	300
0	0	1	0	200	1	0	1	0	150
0	0	1	1	600	1	0	1	1	850
0	1	0	0	300	1	1	0	0	200
0	1	0	1	300	1	1	0	1	250
0	1	1	0	200	1	1	1	0	100
0	1	1	1	700	1	1	1	1	950

Referring to expectations, the model consists of two stochastic functions:

$$\begin{aligned} \sigma_1 &: \ (x,z) \longrightarrow \mathrm{E}(\dot{Y}_1 | \ddot{X}=x, \ddot{Z}=z) \\ \sigma_2 &: \ (x,z,y) \longrightarrow \mathrm{E}(\dot{Y}_2 | \ddot{X}=x, \ddot{Z}=z, \dot{Y}_1=y) \end{aligned}$$

The first function is intended to show how the expectation of the endogenous variable $\dot{Y}_1$ (marked by a single dot) depends on the two exogenous explanatory variables (marked by two dots), the second function is intended to show how the expectation of $\dot{Y}_2$ depends on the same variables and additionally on the outcome of the first learning frame which is used as an endogenous explanatory variable.

2. Conditional and unconditional effects. Without presupposing any particular parametric model, effects of $\ddot{X}$ (SES) on the first outcome can be defined by

$$\Delta^s(\dot{Y}_1; \ddot{X}[x', x''], \ddot{Z}=z) := \mathrm{E}(\dot{Y}_1 | \ddot{X}=x'', \ddot{Z}=z) - \mathrm{E}(\dot{Y}_1 | \ddot{X}=x', \ddot{Z}=z) \tag{1}$$

Effects of $\ddot{X}$ on the second outcome can be defined in two different ways. One can define *conditional effects* which take into account outcomes of the first learning frame:

$$\begin{aligned} &\Delta^s_c(\dot{Y}_2; \ddot{X}[x', x''], \ddot{Z}=z, \dot{Y}_1=y) := \\ &\qquad \mathrm{E}(\dot{Y}_2 | \ddot{X}=x'', \ddot{Z}=z, \dot{Y}_1=y) - \mathrm{E}(\dot{Y}_2 | \ddot{X}=x', \ddot{Z}=z, \dot{Y}_1=y) \end{aligned} \tag{2}$$

And one can consider *unconditional effects*:

$$\Delta^s_u(\dot{Y}_2; \ddot{X}[x', x''], \ddot{Z}=z) := \mathrm{E}(\dot{Y}_2 | \ddot{X}=x'', \ddot{Z}=z) - \mathrm{E}(\dot{Y}_2 | \ddot{X}=x', \ddot{Z}=z) \tag{3}$$

In general, all these effects are context-dependent, that is, depend on values of $\ddot{Z}$. To illustrate, I use the fictitious data in Table 1:

Effects on first outcome:

$$\begin{aligned} \Delta^s(\dot{Y}_1; \ddot{X}[0,1], \ddot{Z}=0) &= 0.667 - 0.533 = 0.134 \\ \Delta^s(\dot{Y}_1; \ddot{X}[0,1], \ddot{Z}=1) &= 0.700 - 0.600 = 0.100 \end{aligned}$$

meaning that higher SES has a positive effect on the educational outcome, but the effect is lower for individuals with a high educational aspiration.

Conditionals effect on second outcome:

$$\begin{aligned} \Delta^s_c(\dot{Y}_2; \ddot{X}[0,1], \ddot{Z}=0, \dot{Y}_1=0) &= 0.600 - 0.571 = 0.029 \\ \Delta^s_c(\dot{Y}_2; \ddot{X}[0,1], \ddot{Z}=1, \dot{Y}_1=0) &= 0.556 - 0.500 = 0.056 \end{aligned}$$

$$\Delta_c^s(\dot{Y}_2; \ddot{X}[0,1], \ddot{Z}=0, \dot{Y}_1=1) = 0.850 - 0.750 = 0.100$$
$$\Delta_c^s(\dot{Y}_2; \ddot{X}[0,1], \ddot{Z}=1, \dot{Y}_1=1) = 0.905 - 0.778 = 0.127$$

Unconditional effect on second outcome:

$$\Delta_u^s(\dot{Y}_2; \ddot{X}[0,1], \ddot{Z}=0) = 0.767 - 0.667 = 0.100$$
$$\Delta_u^s(\dot{Y}_2; \ddot{X}[0,1], \ddot{Z}=1) = 0.800 - 0.667 = 0.133$$

In order to understand the difference between conditional and unconditional effects, it is important to recognize that both concern the outcome in the second learning frame (σ_2). $\dot{Y}_1$ is to be understood as representing a condition for the outcome generation in the second learning frame. This is made explicit in the conditional effect definition. In contrast, the unconditional effect is derived from a reduced model that integrates over the distribution of $\dot{Y}_1$. This is possible, without additional assumptions, because $\dot{Y}_1$ is a mediating variable in the two-stage model:

$$\begin{aligned} &\mathrm{E}(\dot{Y}_2 \,|\, \ddot{X}=x, \ddot{Z}=z) = \\ &\quad \mathrm{E}(\dot{Y}_2 \,|\, \ddot{X}=x, \ddot{Z}=z, \dot{Y}_1=0)\, \mathrm{Pr}(\dot{Y}_1=0 | \ddot{X}=x, \ddot{Z}=z) + \\ &\quad \mathrm{E}(\dot{Y}_2 \,|\, \ddot{X}=x, \ddot{Z}=z, \dot{Y}_1=1)\, \mathrm{Pr}(\dot{Y}_1=1 | \ddot{X}=x, \ddot{Z}=z) \end{aligned} \tag{4}$$

In this sense, viewing $\dot{Y}_1$ as a mediating variable, the unconditional effect corresponds to the total effect of $\ddot{X}$ in the context given by $\ddot{Z}$.

3. Effects of endogenous educational outcomes. So far I have considered effects of an exogenous variable, $\ddot{X}$. In formally the same way one could define effects of $\ddot{Z}$ and use $\ddot{X}$ as a context variable. In order to consider effects of the endogenous variable $\dot{Y}_1$, one has to assume that values of both exogenous variables can be fixed. An effect can then be defined by

$$\begin{aligned} &\Delta^s(\dot{Y}_2; \dot{Y}_1[y', y''], \ddot{X}=x, \ddot{Z}=z) := \\ &\qquad \mathrm{E}(\dot{Y}_2 \,|\, \dot{Y}_1=y'', \ddot{X}=x, \ddot{Z}=z) - \mathrm{E}(\dot{Y}_2 \,|\, \dot{Y}_1=y', \ddot{X}=x, \ddot{Z}=z) \end{aligned} \tag{5}$$

Like the conditional effects of exogenous variables, this effect concerns the generation of values of $\dot{Y}_2$ in the second learning frame. The effect corresponds to the hypothesis that $\dot{Y}_2$ depends on what has been learned in the preceding learning frame even if the context is fixed by particular values of $\ddot{X}$ and $\ddot{Z}$. With the data in Table 1 one finds:

$$\Delta^s(\dot{Y}_2; \dot{Y}_1[0,1], \ddot{X}=0, \ddot{Z}=0) = 0.750 - 0.571 = 0.179$$
$$\Delta^s(\dot{Y}_2; \dot{Y}_1[0,1], \ddot{X}=0, \ddot{Z}=1) = 0.778 - 0.500 = 0.278$$
$$\Delta^s(\dot{Y}_2; \dot{Y}_1[0,1], \ddot{X}=1, \ddot{Z}=0) = 0.850 - 0.600 = 0.250$$
$$\Delta^s(\dot{Y}_2; \dot{Y}_1[0,1], \ddot{X}=1, \ddot{Z}=1) = 0.905 - 0.556 = 0.349$$

showing how the effects of previous learning depend on both $\ddot{X}$ (SES) and $\ddot{Z}$ (aspiration).

4. Endogenously generated inequality. In Model 1, one can think that part of the variation of $\dot{Y}_2$ is due to endogenously generated variation of $\dot{Y}_1$, that is, variation conditional on values of the exogenous variables, $\ddot{X}$ and $\ddot{Z}$. Formally, one can use a conditional variance decomposition

$$\mathrm{V}(\dot{Y}_2 | x, z) = \mathrm{V}[\mathrm{E}(\dot{Y}_2 | \dot{Y}_1, x, z)] + \mathrm{E}[\mathrm{V}(\dot{Y}_2 | \dot{Y}_1, x, z)]$$

This is formally analogue to the variance decomposition discussed in Rohwer (2012: 11f). The first part,

$$\mathrm{V}[\mathrm{E}(\dot{Y}_2 | \dot{Y}_1, x, z)] = \sum\nolimits_{y_1} [\mathrm{E}(\dot{Y}_2 | \dot{Y}_1=y_1, x, z) - \mathrm{E}(\dot{Y}_2 | x, z)]^2\, \mathrm{Pr}(\dot{Y}_1=y_1 | x, z)$$

is now interpreted as the part of the variation of $\dot{Y}_2$ which is generated by variation of $\dot{Y}_1$. The second part,

$$\mathrm{E}[\mathrm{V}(\dot{Y}_2|\dot{Y}_1, x, z)] = \sum\nolimits_{y_1} \mathrm{V}(\dot{Y}_2|\dot{Y}_1 = y_1, x, z)\,\Pr(\dot{Y}_1 = y_1|x, z)$$

is the 'residual variation' which cannot be attributed to $\dot{Y}_1$. Using the data in Table 1, one finds the following values.

x	z	$\mathrm{V}(\dot{Y}_2|x,z)$	$\mathrm{V}[\mathrm{E}(\dot{Y}_2|\dot{Y}_1,x,z)]$	$\mathrm{E}[\mathrm{V}(\dot{Y}_2|\dot{Y}_1,x,z)]$
0	0	0.2222	0.0079	0.2143
0	1	0.2222	0.0185	0.2037
1	0	0.1789	0.0139	0.1650
1	1	0.1600	0.0256	0.1344

In this example, the proportion of variation of $\dot{Y}_2$ which is endogenously generated by variation in the educational outcomes of the first learning frame $(\dot{Y}_1|x, z)$, is between 3.5 and 16 %, depending on the values of the exogenous variables.

5. Consequences of omitted variables. I now consider consequences of omitting an explanatory variable. The general strategy is to compare a hypothetically complete model (here I use Model 1 for illustrations) with a reduced model that results from integrating over the conditional distribution of the omitted variable. There are then two cases.

The first case occurs if the omitted variable is a mediating variable in the complete model. Its conditional distribution can then be derived from the model without further assumptions. Referring to Model 1, this case is illustrated by omitting $\dot{Y}_1$ (see § 1.2).

The second case occurs if the conditional distribution of the omitted variable cannot be derived from the complete model without further assumptions. To illustrate, I assume that $\ddot{Z}$ is omitted and one is interested in effects of $\ddot{X}$. In order to derive a reduced model, $\ddot{Z}$ must be substituted by a variable, say $\dot{Z}$, that can be assumed to have a (conditional) distribution. The derivation depends on the kind of effect.

I begin with the effect of $\ddot{X}$ on $\dot{Y}_1$. For the reduced model, one immediately finds

$$\mathrm{E}(\dot{Y}_1|\ddot{X} = x) = \sum\nolimits_z \mathrm{E}(\dot{Y}_1|\ddot{X} = x, \dot{Z} = z)\,\Pr(\dot{Z} = z|\ddot{X} = x) \tag{6}$$

A special case occurs if the effect of $\ddot{X}$ on $\dot{Y}_1$ does not depend on values of $\dot{Z}$. Then, omitting $\dot{Z}$ does not change the relationship between $\ddot{X}$ and $\dot{Y}_1$.[1] In general, the effect of $\ddot{X}$ in the reduced model is a kind of mean value of its context-dependent effects in the complete model.

The same considerations apply to the unconditional effect of $\ddot{X}$ on $\dot{Y}_2$ for which one gets the reduced expression

$$\mathrm{E}(\dot{Y}_2|\ddot{X} = x) = \sum\nolimits_z \mathrm{E}(\dot{Y}_2|\ddot{X} = x, \dot{Z} = z)\,\Pr(\dot{Z} = z|\ddot{X} = x) \tag{7}$$

which is completely analogue to (6).

When considering the conditional effect of $\ddot{X}$ on $\dot{Y}_2$ one has to take into account that $\ddot{Z}$ is a confounder w.r.t. the relationship between $\dot{Y}_1$ and $\dot{Y}_2$. The reduced relationship is given by

$$\mathrm{E}(\dot{Y}_2|\ddot{X} = x, \dot{Y}_1 = y_1) = \sum\nolimits_z \mathrm{E}(\dot{Y}_2|\ddot{X} = x, \dot{Z} = z, \dot{Y}_1 = y_1)\,\Pr(\dot{Z} = z|\ddot{X} = x, \dot{Y}_1 = y_1) \tag{8}$$

[1] Note that it is not essential whether $\dot{Z}$ depends on $\ddot{X}$; the essential point concerns whether there is an interaction between $\ddot{X}$ and $\ddot{Z}$ w.r.t. $\dot{Y}_1$.

Here the conditional distribution of $\dot{Z}$ that is used for mixing the conditional effects no longer only depends on exogenous variables ($\ddot{X}$ in this example), but also on the endogenous variable $\dot{Y}_1$. In general, this also changes the relationship between $\dot{Z}$ and $\ddot{X}$. For example, even if $\dot{Z}$ is independent of $\ddot{X}$, this is no longer true conditional on values of $\dot{Y}_1$. (This is illustrated by the data in Table 1.) Whether this provides a reason for thinking that the effect of $\ddot{X}$ in the reduced model is in some sense 'biased' will be discussed in § 1.7.

6. Example with linear regression functions. For further illustration of consequences of omitted variables, I still refer to Model 1 but assume linear functional relationships:

$$\mathrm{E}(\dot{Y}_1|x,z) = \alpha_1 + x\beta_1 + z\gamma_1 \tag{9}$$

$$\mathrm{E}(\dot{Y}_2|x,z,y_1) = \alpha_2 + x\beta_2 + z\gamma_2 + y_1\delta \tag{10}$$

As before, the interest concerns effects of x when z is omitted. In order to derive reduced models, I assume that the distribution of the values of z is given by a density function $f(z|x)$ with mean μ_z which, for simplicity, does not depend on x.

(a) I begin with the effect of x on the expectation of $\dot{Y}_1$. The relationship in the reduced model is given by

$$\mathrm{E}(\dot{Y}_1|x) = \int_z \mathrm{E}(\dot{Y}_1|x,z)\, f(z|x)\, dz$$

and one immediately finds

$$\mathrm{E}(\dot{Y}_1|x) = (\alpha_1 + \mu_z\gamma_1) + x\beta_1$$

showing that the parameter associated with x does not change. (This would not be true if (9) contained an interaction between x and z.)

(b) I now consider unconditional effects of x and z on $\dot{Y}_2$. The relationship in the reduced model is then given by

$$\mathrm{E}(\dot{Y}_2|x,z) = \int_{y_1} \mathrm{E}(\dot{Y}_2|x,z,y_1)\, f(y_1|x,z)\, dy_1$$

where $f(y_1|x,z)$ is the conditional density of $\dot{Y}_1$. Entailed by (9), this is a density with mean $\alpha_1 + x\beta_1 + z\gamma_1$. It follows that

$$\mathrm{E}(\dot{Y}_2|x,z) = (\alpha_2 + \alpha_1\delta) + x\,(\beta_2 + \beta_1\delta) + z\,(\gamma_2 + \gamma_1\delta) \tag{11}$$

The parameters are obviously different from the original parameters in (10). However, they are not 'biased', but correctly express the total (unconditional) effects of x and z on the expectation of $\dot{Y}_2$.

(c) I now consider the unconditional effect of x on $\dot{Y}_2$ when z is omitted. The relationship in the reduced model is then given by

$$\mathrm{E}(\dot{Y}_2|x) = \int_{y_1,z} \mathrm{E}(\dot{Y}_2|x,z,y_1)\, f(y_1,z|x)\, dy_1\, dz$$

where $f(y_1,z|x)$ is the joint density of y_1 and z. Since $f(y_1,z|x) = f(y_1|x,z)\, f(z|x)$, one can continue with the derivation in (b):

$$\mathrm{E}(\dot{Y}_2|x) = \int_z \mathrm{E}(\dot{Y}_2|x,z)\, f(z|x)\, dz = \alpha_2 + \alpha_1\delta + \mu_z\,(\gamma_2 + \gamma_1\delta) + x\,(\beta_2 + \beta_1\delta) \tag{12}$$

The parameter associated with x is the same as in (11) where z was not omitted and has the same interpretation.

(d) Finally, I consider the conditional effect of x on $\dot{Y}_2$ when z is omitted. The relationship in the reduced model is then given by

$$\mathrm{E}(\dot{Y}_2|x,y_1) = \int_z \mathrm{E}(\dot{Y}_2|x,z,y_1)\, f(z|x,y_1)\, dz = \alpha_2 + x\,\beta_2 + y_1\delta + \mathrm{E}(\dot{Z}|x,y_1)\,\gamma_2 \tag{13}$$

where $f(z|x,y_1)$ is the density of z conditional on x and y_1, and

$$\mathrm{E}(\dot{Z}|x,y_1) = \int_z z f(z|x,y_1)\, dz$$

is the conditional mean of z. How parameters change is best seen when using a linear relationship

$$\mathrm{E}(\dot{Z}|x,y_1) = \alpha_z + x\beta_z + y_1\delta_z \tag{14}$$

Inserting this into (13) leads to

$$\mathrm{E}(\dot{Y}_2|x,y_1) = (\alpha_2 + \alpha_z\gamma_2) + x\,(\beta_2 + \beta_z\gamma_2) + y_1\,(\delta + \delta_z\gamma_2) \tag{15}$$

Parameters now consist of two parts. A first part equals the parameters as presupposed in (10), a second part consists of the contribution to estimating the conditional mean of the omitted variable, $\mathrm{E}(\dot{Z}|x,y_1)$.

To illustrate, assume $\alpha_1 = 1$, $\beta_1 = 1$, $\gamma_1 = 1$, $\alpha_2 = 0.5$, $\beta_2 = 1.5$, $\gamma_2 = 2$, $\delta = 0.2$, and $\mu_z = 1$. One then finds: $\alpha_z = 0$, $\beta_z = -0.5$, $\delta_z = 0.5$, entailing that the coefficient of x in (15) is less than β_2: $\beta_2 + \beta_z\gamma_2 = 1.5 - 0.5 \cdot 2$, and the coefficient of y_1 is greater than δ: $\delta + \delta_z\gamma_2 = 0.2 + 0.5 \cdot 2$.

Of course, there is nothing wrong with these parameters. Assume that one knows an individual's values of $\ddot{X}$ and $\dot{Y}_1$, but not of $\ddot{Z}$. In order to apply the original model (10), one would need an estimate of the individual's value of $\ddot{Z}$. If one uses the conditional expectation as defined in (14), then (10) and (15) lead to the same result. In this sense, the difference in the parameters is required in order to get correct estimates with the reduced model.

7. Are effect definitions 'biased'? When compared with (10), it is tempting to say that conditional effects calculated with (15) are 'biased'. However, one should be clear about what this possibly could mean.

(a) Not only the estimation, already the definition of an effect must be based on assuming a particular model. For example, one can use the following

Model 2

$$\begin{tikzcd}
\ddot{X} \arrow[d] \arrow[dr] & \\
\dot{Y}_1 \arrow[r] & \dot{Y}_2
\end{tikzcd}$$

with the same variables as used for Model 1 to *define* an effect

$$\Delta^s(\dot{Y}_2; \ddot{X}[x',x''], \dot{Y}_1 = y) := \mathrm{E}(\dot{Y}_2|\,\ddot{X} = x'', \dot{Y}_1 = y) - \mathrm{E}(\dot{Y}_2|\,\ddot{X} = x', \dot{Y}_1 = y) \tag{16}$$

This effect compares the outcomes, in the second learning frame, $\dot{Y}_2$, of two *generic* individuals, one with $\ddot{X} = x'$ and the other one with $\ddot{X} = x''$. As made explicit in the definition, both have the same educational outcome, $\dot{Y}_1 = y$, in the first learning frame. However, no further assumptions are made about the individuals; they can differ in all variables not explicitly represented in the model (e.g. $\ddot{Z}$).[2]

(b) The statement that the effect defined in (16) is 'biased' has no meaning without explicitly referring to another model that can be used as a standard of comparison. For example, it might be said that the effect is biased because one *should* use Model 1 to define a 'less biased' effect. Such a normative statement is also required in order to call Model 2 a 'misspecified model'.[3]

(c) It is not possible to find, or generate, data for estimating Model 2 which entail that conditional distributions of the values of omitted variables (e.g. $\ddot{Z}$) are independent of $\ddot{X}$ and $\dot{Y}_1$. In order to assess the theoretically posited bias one would need to actually estimate the more comprehensive Model 1, and this would require to get data about $\ddot{Z}$.

(d) Equation (13) seems to suggest that including estimates of the conditional mean values, $\mathrm{E}(\dot{Z}|x, y_1)$, into the reduced model would allow one to find an 'unbiased' estimate of β_2. This idea was proposed by Heckman (1979) who showed that in a class of specifically parameterized linear models one can find estimates of the conditional mean values of $\dot{Z}$ without observing its values. However, the required assumptions are very restrictive, and it has been shown that the approach can easily lead to very misleading parameter estimates (LaLonde 1986, Briggs 2004).

(e) It is a quite specific property of the simple linear model (10) that effects of $\ddot{X}$, $\ddot{Z}$ and $\dot{Y}_1$ are context-independent. In general, there are no context-independent effects to which effects derived from reduced models can be compared. In other words, then, already the presupposition that an effect can be associated uniquely with a single variable is misleading.

2. Sequential transitions

1. Sequential binary choices. I now turn to models of sequential transitions. I begin with considering three consecutive learning frames, σ_j ($j = 1, 2, 3$), and two binary variables, $\dot{T}_1$ and $\dot{T}_2$: $\dot{T}_j = 1$ if there is a transition from σ_j to σ_{j+1}, and $\dot{T}_j = 0$ if there is no transition into another learning frame. I consider the following

Model 3

$$\begin{tikzcd}
\ddot{X} \arrow[d] \arrow[dr] & \\
\dot{T}_1 \arrow[r] & \dot{T}_2 \\
\ddot{Z} \arrow[u] \arrow[ur] &
\end{tikzcd}$$

The model has the same structure as Model 1, with $\dot{T}_j$ instead of $\dot{Y}_j$, but there is now the constraint

$$\Pr(\dot{T}_2 = 1 \,|\, \ddot{X} = x, \ddot{Z} = z, \dot{T}_1 = 0) = 0 \tag{17}$$

[2]This understanding of 'functional effects' is therefore quite different from effect definitions in the potential outcomes framework which are conceptually linked to particular individuals. For further discussion see Rohwer (2012, Sec. 3).

[3]This formulation is used, e.g., by Cameron and Heckman (1998).

Table 2 Fictitious data for the sequential transition model.

x	z	t_1	t_2	cases	x	z	t_1	t_2	cases
0	0	0	0	500	1	0	0	0	300
0	0	0	1	0	1	0	0	1	0
0	0	1	0	200	1	0	1	0	140
0	0	1	1	300	1	0	1	1	560
0	1	0	0	400	1	1	0	0	100
0	1	0	1	0	1	1	0	1	0
0	1	1	0	200	1	1	1	0	90
0	1	1	1	400	1	1	1	1	810

and from this follows

$$\mathrm{E}(\dot{T}_2 \mid \ddot{X}=x, \ddot{Z}=z) = \mathrm{E}(\dot{T}_2 \mid \ddot{X}=x, \ddot{Z}=z, \dot{T}_1=1)\, \mathrm{E}(\dot{T}_1 \mid \ddot{X}=x, \ddot{Z}=z) \tag{18}$$

2. Consideration of possible effects. In the same way as was discussed in the previous section, one can distinguish conditional and unconditional effects of the exogenous variables on $\dot{T}_2$. For example, the conditional effect of $\ddot{X}$ is

$$\begin{aligned}&\Delta_c^s(\dot{T}_2; \ddot{X}[x', x''], \ddot{Z}=z, \dot{T}_1=1) := \\ &\qquad \mathrm{E}(\dot{T}_2 \mid \ddot{X}=x'', \ddot{Z}=z, \dot{T}_1=1) - \mathrm{E}(\dot{T}_2 \mid \ddot{X}=x', \ddot{Z}=z, \dot{T}_1=1)\end{aligned} \tag{19}$$

and the unconditional effect is

$$\Delta_u^s(\dot{T}_2; \ddot{X}[x', x''], \ddot{Z}=z) := \mathrm{E}(\dot{T}_2 \mid \ddot{X}=x'', \ddot{Z}=z) - \mathrm{E}(\dot{T}_2 \mid \ddot{X}=x', \ddot{Z}=z) \tag{20}$$

In general, both conditional and unconditional effects are context-dependent, that is, depend on values of $\ddot{Z}$. To illustrate, I use the fictitious data in Table 2:

Effects on first transition:

$\Delta^s(\dot{T}_1; \ddot{X}[0,1], \ddot{Z}=0) = 0.70 - 0.50 = 0.20$
$\Delta^s(\dot{T}_1; \ddot{X}[0,1], \ddot{Z}=1) = 0.90 - 0.60 = 0.30$

Conditionals effect on second transition:

$\Delta_c^s(\dot{T}_2; \ddot{X}[0,1], \ddot{Z}=0, \dot{T}_1=1) = 0.80 - 0.60 = 0.20$
$\Delta_c^s(\dot{T}_2; \ddot{X}[0,1], \ddot{Z}=1, \dot{T}_1=1) = 0.90 - 0.67 = 0.23$

Unconditional effect on second transition:

$\Delta_u^s(\dot{T}_2; \ddot{X}[0,1], \ddot{Z}=0) = 0.56 - 0.30 = 0.26$
$\Delta_u^s(\dot{T}_2; \ddot{X}[0,1], \ddot{Z}=1) = 0.81 - 0.40 = 0.41$

How to think of possible effects of $\dot{T}_1$? These effects are already defined by the set-up of the model and need not to be estimated. $\dot{T}_1 = 0$ deterministically entails $\dot{T}_2 = 0$. $\dot{T}_1 = 1$, on the other hand, is a necessary condition for $\dot{T}_2 = 1$, but has no further causal effect on the probability distribution of $\dot{T}_2$. The role played by $\dot{T}_1$ in Model 3 is therefore quite different from the role played by $\dot{Y}_1$ in Model 1.

3. Models in terms of latent variables. Parametric models for the transition variables, $\dot{T}_j$, are often set up in terms of latent variables. This allows formulations which, in a sense, parallel

(9) and (10), most easily in pseudo-indeterministic forms (based on assuming 'residual' random variables). For example:

$$\dot{T}_1^* = \alpha_1 + x\beta_1 + z\gamma_1 + \dot{U}_1 \quad (21)$$

$$\dot{T}_2^* = \alpha_2 + x\beta_2 + z\gamma_2 + \dot{U}_2 \quad (22)$$

$\dot{T}_1^*$ and $\dot{T}_2^*$ are the latent variables which are linked to the binary transition variables by $\dot{T}_1 = I[\dot{T}_1^* \geq 0]$ and $\dot{T}_2 = I[\dot{T}_2^* \geq 0]$. $\dot{U}_1$ and $\dot{U}_2$ are random variables with distribution functions F_1 and F_2, respectively. The distributions are most often assumed to be symmetrical around a zero mean. This then entails:

$$\mathrm{E}(\dot{T}_1 | x, z) = F_1(\alpha_1 + x\beta_1 + z\gamma_1) \quad (23)$$

$$\mathrm{E}(\dot{T}_2 | x, z, \dot{T}_1 = 1) = F_2(\alpha_2 + x\beta_2 + z\gamma_2) \quad (24)$$

showing that F_1 and F_2 simply serve as functions linking the explanatory variables to the binomial distributions of the outcome variables.

However, even when using latent variables there remains an essential difference between models 1 and 3. Model 1 relates to a generic individual that participates in both learning frames, and this allows one to think of a joint distribution of the two educational outcomes, $\dot{Y}_1$ and $\dot{Y}_2$. Now consider the sequential transition model 3. In this model, $\dot{T}_1 = 0$ deterministically entails $\dot{T}_2 = 0$. While one can nevertheless think of a joint distribution of $\dot{T}_1$ and $\dot{T}_2$, there is no corresponding joint distribution of the latent variables, $\dot{T}_1^*$ and $\dot{T}_2^*$. If $\dot{T}_1^* < 0$, there is no transition and consequently no second latent variable, $\dot{T}_2^*$.

The argument shows that model specifications for the sequential transition model cannot sensibly begin with assuming a joint distribution for latent transition variables.[4]

4. Describing selection processes. In the sequential transition model, the transition variables can be interpreted as representing selections: if $\dot{T}_j = 1$, an individual is selected into a subsequent learning frame. However, in order to actually consider a selection process one needs a collection of individuals, say Ω, and statistical variables corresponding to the model's exogenous variables. As an example, one can use the data in Table 2 which provide values for

$$(X, Z, T_1, T_2) : \Omega \longrightarrow \{0, 1\}^4 \quad (25)$$

Based on information about these variables one can describe how the distributions of the exogenous variables change in the consecutive learning frames. In this example:

	t_1	t_2	$\mathrm{P}(X=1 \mid T_1=t_1, T_2=t_2)$	$\mathrm{P}(Z=1 \mid T_1=t_1, T_2=t_2)$
σ_1	0	0	0.308	0.385
σ_2	1	0	0.365	0.460
σ_3	1	1	0.662	0.585

Assuming that $X = 1$ represents 'high SES', and $Z = 1$ 'high aspiration', this would mean that the selection process leads to larger proportions of individuals with 'high SES' and 'high aspiration'.

In order to formulate a general relationship between effects on transitions and subsequent selections, I consider the unconditional effects

$$\Delta_u^s(\dot{T}_j; \ddot{X}[x', x''], \ddot{Z} = z) := \mathrm{E}(\dot{T}_j \mid \ddot{X} = x'', \ddot{Z} = z) - \mathrm{E}(\dot{T}_2 \mid \ddot{X} = x', \ddot{Z} = z) \quad (26)$$

[4] As an example, I refer to Holm and Jaeger (2011) who proposed to begin with assuming a joint normal distribution for the latent transition variables.

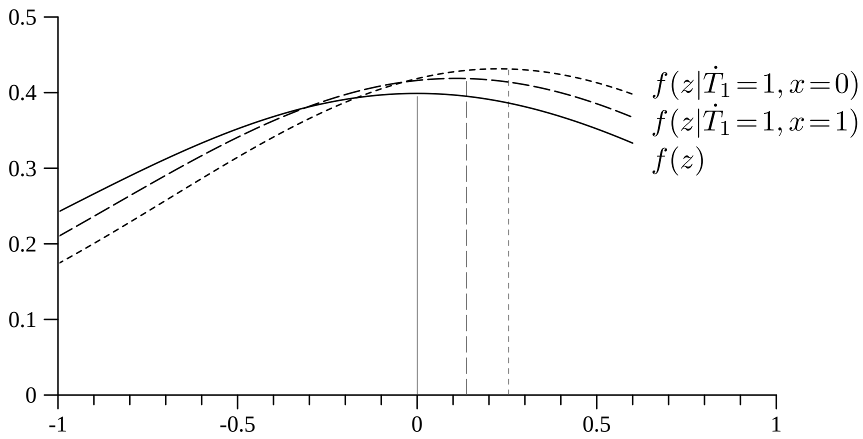


Figure 1 Unconditional and conditional density functions of Z and their mean values as described in the text.

Given corresponding statistical variables, as defined in (25), there is the following simple relationship:

$$\Delta_u^s(\dot{T}_j; \ddot{X}[x', x''], \ddot{Z}=z) > 0 \iff \frac{\mathrm{P}(X=x''|T_j=1, Z=z)}{\mathrm{P}(X=x'|T_j=1, Z=z)} > \frac{\mathrm{P}(X=x''|Z=z)}{\mathrm{P}(X=x'|Z=z)}$$

This is easily understandable: If individuals with $\ddot{X}=x''$ have a higher probability for making the transition $T_j = 1$ than individuals with $\ddot{X} = 0$, they will become relatively more frequent conditional on $T_j=1$.

Note, however, that this relationship has no implications for a comparison of effects *across* transitions. In general, the conditional effect defined in (19) can be greater or smaller than the effect of $\ddot{X}$ on the first transition.

5. Consequences of omitted variables. Also in the sequential transition model one can think about omitted variables. This is often a concern in the literature.[5] Considerations parallel the discussion of omitted variables in the models of sequential educational outcomes, and the results of § 1.5 and the conclusions of § 1.7 can be applied without any essential modification.

It might nevertheless be interesting to consider a further example. For this example, I start from (23) and (24) and assume logit specifications. Corresponding to $\ddot{X}$ I assume a binary variable, X, with $\mathrm{P}(X=1) = 0.5$; and corresponding to $\ddot{Z}$ I assume a continuous variable, Z, with a standard normal density, $f(z)$, independent of X. For the parameters I choose $\alpha_1 = \beta_1 = \gamma_1 = 1$ and $\alpha_2 = \beta_2 = \gamma_2 = 1$.

Conditional on $T_1=1$ both distributions change. Instead of $\mathrm{P}(X=1) = 0.5$, there is a higher value $\mathrm{P}(X = 1|T_1 = 1) = 0.55$. How the distribution of Z changes is shown in Figure 1. The figure also shows that X and Z are no longer independent. Both conditional mean values have increased: $\mathrm{M}(Z|T_1=1, X=0) = 0.255$, and $\mathrm{M}(Z|T_1=1, X=1) = 0.137$. The difference is due to a negative correlation between X and Z which, in this example, results from conditioning on T_1.

Now assume that Z is not observed. The conditional effect of $\ddot{X}$ on the second transition that

[5] E.g., Mare (1993), Cameron and Heckman (1998), Holm and Jaeger (2011).

can be estimated is then given by

$$\begin{aligned}
&\mathrm{E}(\dot{T}_2|\ddot{X}=1,\dot{T}_1=1) - \mathrm{E}(\dot{T}_2|\ddot{X}=0,\dot{T}_1=1) = \\
&\quad \int_z \mathrm{E}(\dot{T}_2|\ddot{X}=1,\dot{Z}=z,\dot{T}_1=1)\, f(z|\ddot{X}=1,\dot{T}_1=1)\, dz - \\
&\qquad \int_z \mathrm{E}(\dot{T}_2|\ddot{X}=0,\dot{Z}=z,\dot{T}_1=1)\, f(z|\ddot{X}=0,\dot{T}_1=1)\, dz
\end{aligned} \tag{27}$$

In our example, this effect has the value

$$0.1185 = 0.8630 - 0.7445$$

Of course, one can argue that this effect is not only due to $\ddot{X}$ but also to the selection effect that shows up in the different conditional distributions of $\dot{Z}$. In our example, these effects can be separated by using $f(z|x)$ instead of $f(z|x,\dot{T}_1=1)$. Instead of (27), one then calculates a counterfactual effect with

$$\int_z \mathrm{E}(\dot{T}_2|\ddot{X}=1,\dot{Z}=z,\dot{T}_1=1)\, f(z|\ddot{X}=1)\, dz - \int_z \mathrm{E}(\dot{T}_2|\ddot{X}=0,\dot{Z}=z,\dot{T}_1=1)\, f(z|\ddot{X}=0)\, dz \tag{28}$$

In our example, one finds the value

$$0.1478 = 0.8445 - 0.6967$$

which is obviously larger. The difference can be attributed to the selection effect.

However, it would not be sensible to think of the effect calculated with (27) as a 'biased estimate' of the effect defined by (28). The latter effect is purely counterfactual, and could only be calculated if $\ddot{Z}$ had been observed.

3. Primary and secondary effects

The models discussed in Section 2 make transition probabilities directly dependent on variables representing the family background (e.g. parents' SES and educational levels). It is highly plausible, however, that transition probabilities also depend on the educational outcomes in earlier learning frames. This idea has lead to a longstanding debate about how effects of the family background are mediated.

Following Boudon (1974), researchers often focus on a distinction between primary and secondary effects (see, e.g., Baumert et al. 2003, Erikson et al. 2005, Jackson et al. 2007, Erikson and Rudolphi 2010, Schindler and Reimer 2010, Neugebauer 2010). I take up this discussion in the present section and consider two different modeling approaches. Then, in Section 4, I discuss an enlarged version of the sequential transition model of Section 2 in which a distinction between primary and secondary effects is explicitly represented.

1. An often used modeling approach. Researchers often use a model that basically has the following structure:

Model 4

$$\begin{tikzcd}
\ddot{X} \arrow[r] \arrow[d] & \dot{T} \\
\dot{Z} \arrow[ur] &
\end{tikzcd}$$

$\dot{T}$ is a binary dependent variable representing the transition of interest, for example, $\dot{T}=1$ if there is a transition from lower to higher secondary education, and $\dot{T} = 0$ otherwise. There are two main explanatory variables:

$\ddot{X}$ is an exogenous explanatory variable that represents some aspect of the family background, e.g., fathers occupational class (Erikson et al. 2005), or parents' educational level (Kloosterman et al. 2009, Neugebauer 2010).

$\dot{Z}$ is an endogenous explanatory variables which represents the 'level of academic performance' an individual has reached in the situation where the transition is to be made. Values are often based on a scholastic aptitude test.

This model is then used to distinguish two kinds of effects of $\ddot{X}$. A *primary effect* concerns the dependence of $\dot{Z}$ on $\ddot{X}$. For example, comparing x' and x'', the primary effect could be defined as

$$\mathrm{E}(\dot{Z}|\ddot{X}=x'') - \mathrm{E}(\dot{Z}|\ddot{X}=x')$$

The *secondary effect* concerns the effect of $\ddot{X}$ on $\dot{T}$, while $\dot{Z}$ is in some way fixed. For example,

$$\Pr(\dot{T}=1|\ddot{X}=x'',\dot{Z}=z) - \Pr(\dot{T}=1|\ddot{X}=x',\dot{Z}=z)$$

2. Representing transition opportunities. Model 4 presupposes a direct link between 'academic performance' and the transition variable. A different modeling approach attempts to represent the individual's choice situation. I assume that this can be done with a binary variable, say $\dot{C}$, such that $\dot{C}=1$ if an individual has the choice to opt for the transition of interest. So one can distinguish between (a) the dependence of $\dot{C}$ on $\ddot{X}$ ('primary effect'), and (b) the dependence of $\dot{T}$ on $\ddot{X}$ given that $\dot{C}=1$ ('secondary effect'). Assuming that $\dot{C}$ depends in some way on 'academic performance', $\dot{Z}$, leads one to the following model:

Model 5

$$\begin{tikzcd}
\ddot{X} \arrow[r, twoheadrightarrow] \arrow[d] & \dot{T} \\
\dot{Z} \arrow[r] & \dot{C} \arrow[u, twoheadrightarrow]
\end{tikzcd}$$

Instead of a direct link between $\dot{Z}$ and $\dot{T}$, there is now a mediating binary variable, $\dot{C}$. For subsequent illustrations, I assume a simple deterministic relationship between $\dot{Z}$ and $\dot{C}$:

$$\dot{C} = I[\dot{Z} \geq z_0]$$

where z_0 is a threshold value. $I[\ldots]$ denotes the indicator function; in this example: $I[\dot{Z} \geq z_0]$ takes the value 1 if $\dot{Z} \geq z_0$ and is otherwise zero.

Instead of the definitions given in § 3.1, one now gets quite different expressions for primary and secondary effects. Assuming that $\dot{Z}$ is a discrete variable, the primary effect is given by

$$\Pr(\dot{C}=1|\ddot{X}=x'') - \Pr(\dot{C}=1|\ddot{X}=x') = \sum\nolimits_{z\geq z_0} \big(\Pr(\dot{Z}=z|\ddot{X}=x'') - \Pr(\dot{Z}=z|\ddot{X}=x')\big)$$

And, since the model entails that $\Pr(\dot{T}=1|\dot{C}=0) = 0$, the secondary effect is given by

$$\Pr(\dot{T}=1|\ddot{X}=x'',\dot{C}=1) - \Pr(\dot{T}=1|\ddot{X}=x',\dot{C}=1)$$

Notice that, given $\dot{C}$, this effect is now independent of $\dot{Z}$. (A model that assumes an additional dependence of $\dot{T}$ on $\dot{Z}$ will be considered below.)

Table 3 Values of conditional probabilities assumed in Model 5.

x	μ_x	$\Pr(\dot{C}=1 \mid \ddot{X}=x)$	$\Pr(\dot{T}=1 \mid \dot{C}=1, \ddot{X}=x)$	$\Pr(\dot{T}=1 \mid \ddot{X}=x)$
0	0.00	0.84	0.5	0.42
1	0.05	0.85	0.6	0.52
2	0.10	0.87	0.7	0.60
3	0.15	0.88	0.8	0.70
4	0.20	0.89	0.9	0.80

3. Numerical illustration with simulated data. The two models can lead to quite different conclusions. For illustration, I use Model 5 to create artificial data. $\ddot{X} \in \{0, 1, 2, 3, 4\}$. For $\ddot{X}=x$, $\dot{Z}$ is normally distributed with mean μ_x and variance 1. Threshold value: $z_0 = -1$. Table 3 shows the values assumed for μ_x in column 2, and the assumed conditional probabilities $\Pr(\dot{T}=1 \mid \dot{C}=1, \ddot{X}=x)$ in column 4 (these are the values resulting from the simulation). This allows deriving values for the remaining conditional probabilities. The relationship is given by

$$\Pr(\dot{T}=1|\ddot{X}=x) = \Pr(\dot{T}=1|\dot{C}=1, \ddot{X}=x) \Pr(\dot{C}=1|\ddot{X}=x)$$

Obviously, in this example almost all effects are secondary.

Now I assume Model 4 instead of 5 and use a logit model to specify a continuous dependence of the probability of $\dot{T}=1$ on values of $\dot{Z}$ and dummy variables $X_j := I[\ddot{X}=j]$:

$$\Pr(\dot{T}=1|\ddot{X}=x, \dot{Z}=z) \approx \frac{\exp(X_0\beta_0 + X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + X_4\beta_4 + Z\gamma)}{1 + \exp(X_0\beta_0 + X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + X_4\beta_4 + Z\gamma)}$$

In order to calculate parameters, I use, for each value of $\ddot{X}$, 10000 cases, randomly generated according to the assumptions in Table 3.[6] The estimated parameters are:

$$\hat{\beta}_0 = -0.3696,\ \hat{\beta}_1 = 0.0503,\ \hat{\beta}_2 = 0.3719,\ \hat{\beta}_3 = 0.8352,\ \hat{\beta}_4 = 1.3610,\ \hat{\gamma}\ = 0.7586$$

Figure 2 shows, for $\ddot{X}=1$ and $\ddot{X}=4$, how the estimated probabilities depend on values of $\dot{Z}$.

Compared with Model 5, the results are quite different. In Model 5, if $\dot{Z} \geq z_0$, there is no further effect of $\dot{Z}$ on $\dot{T}$. The logit model, in contrast, suggests that $\dot{Z}$ has always an important effect, even conditional on values of $\ddot{X}$. For example:

$$\Pr(\dot{T}=1|\dot{Z}=1, \ddot{X}=1) - \Pr(\dot{T}=1|\dot{Z}=0, \ddot{X}=1) = 0.2$$
$$\Pr(\dot{T}=1|\dot{Z}=1, \ddot{X}=4) - \Pr(\dot{T}=1|\dot{Z}=0, \ddot{X}=4) = 0.1$$

Correspondingly, the logit model underestimates secondary effects. Consider, for example, the effect

$$\Pr(\dot{T}=1|\dot{Z}=1, \ddot{X}=4) - \Pr(\dot{T}=1|\dot{Z}=1, \ddot{X}=1)$$

While its value is $0.9 - 0.6 = 0.3$ in Model 5, the logit model suggests the value 0.2.

4. Hypothetical effect combinations. In order to illustrate the relative importance of primary and secondary effects, some authors have suggested to consider hypothetical effect combinations (Erikson et al. 2005, Jackson et al. 2007, Kloosterman et al. 2009, Schindler and Reimer 2010). This can easily be done with Model 5. One can calculate

$$H'(x, x') := \Pr(\dot{T}=1 \mid \dot{C}=1, \ddot{X}=x') \Pr(\dot{C}=1 \mid \ddot{X}=x)$$

[6] Individual values of $\dot{T}$, say t_i for an individual i with covariate values x_i and z_i, are generated as follows: $t_i = 1$ if $z_i \geq -1$ and $r_i \leq \Pr(\dot{T}=1 \mid \dot{C}=1, \ddot{X}=x_i)$, where r_i is a random number equally distributed in $[0, 1]$; otherwise $t_i = 0$.

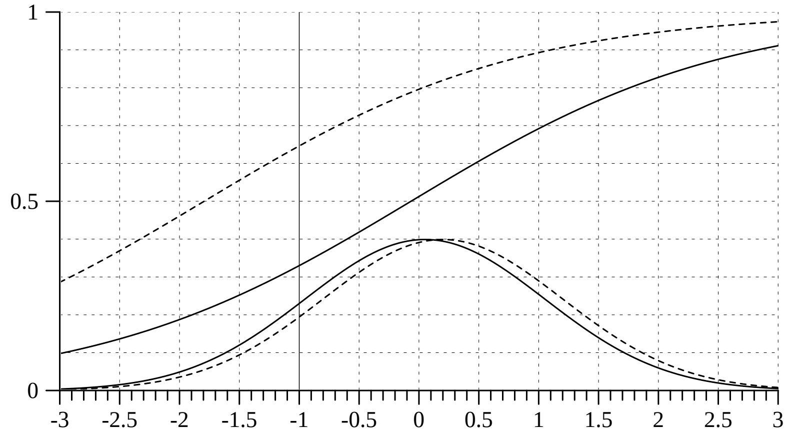


Figure 2 Distribution of $\dot{Z}$ for $\ddot{X}=1$ (solid line) and $\ddot{X}=4$ (dashed line). Also shown: $\exp(\hat{\beta}_1 + z\hat{\gamma})/(1+\exp(\hat{\beta}_1 + z\hat{\gamma}))$ (solid) and $\exp(\hat{\beta}_4 + z\hat{\gamma})/(1+\exp(\hat{\beta}_4 + z\hat{\gamma}))$ (dashed).

and interpret this as the probability of $\dot{T}=1$ under the assumption that the primary effect is given by $\ddot{X}=x$ and the secondary effect is given by $\ddot{X}=x'$.

A similar calculation can be done with Model 4 by exploiting the knowledge of the conditional distributions of $\dot{Z}$. In our example, it was assumed that, depending on $\ddot{X}=x$, $\dot{Z}$ is normally distributed with density $\phi(z;\mu_x,1)$. So one can calculate

$$H(x,x') := \int_z \Pr(\dot{T}=1 \,|\, \ddot{X}=x', \dot{Z}=z)\, \phi(z;\mu_x,1)\, dz \tag{29}$$

and interpret this as the probability of $\dot{T}=1$ under the assumption that the primary effect is given by x and the secondary effect is given by x'.

The quantities $H(x,x')$ and $H'(x,x')$ can be used for a decomposition of the total effect which is analogous to the decomposition into a direct and an indirect effect that was discussed in Rohwer (2012: 16f):

$$H(x'',x'') - H(x',x') = [H(x'',x'') - H(x',x'')] + [H(x',x'') - H(x',x')] \tag{30}$$

(analogous for H'). The first term on the right-hand side is the primary effect that corresponds to the indirect effect of $\ddot{X}$, the second term is the secondary effect that corresponds to the direct effect of $\ddot{X}$.

Values of $H(x,x')$ and $H'(x,x')$ for our example are shown in Table 4. To illustrate, I consider decompositions of $H(4,4) - H(1,1)$:

	primary	secondary	total
Model 4	0.016	0.258	0.274
Model 5	0.036	0.255	0.291

This confirms that in our example the secondary effect is much greater than the primary effect.

5. Comparing the two models. Models 4 and 5 are different in several respects. First, there are different data requirements. Model 4 only requires some measure of 'academic performance'. This can be based on institutionalized scholastic aptitude tests, or tests developed and performed by a researcher. In contrast, the primarily important data requirement for Model 5 concerns the variable $\dot{C}$ which represents the choice situation.

A second point concerns the interpretation. Model 5 is based on a conceptually clear distinction between primary and secondary effects: primary effects concern the generation of opportunities,

Table 4 Hypothetical probabilities of $\dot{T} = 1$ when assuming that primary effects depend on $\ddot{X} = x$ and secondary effects depend on $\ddot{X} = x'$. Calculations based on data in Table 3.

	Model 4		Model 5	
x	$H(x, 1)$	$H(x, 4)$	$H'(x, 1)$	$H'(x, 4)$
0	0.513	0.773	0.425	0.756
1	0.521	0.779	0.510	0.765
2	0.529	0.785	0.595	0.783
3	0.536	0.789	0.680	0.792
4	0.545	0.795	0.765	0.801

secondary effects concern the actually performed choices. There is no correspondingly clear distinction in Model 4. The idea that the chosen transition in some way depends on 'academic performance' is not easily interpretable.

This reflects a deeper distinction between the two models. Model 5 can be understood as an analytical model that aims to understand effects of the family background both on the generation and use of educational transition opportunities. In contrast, Model 4 aims to attribute the actually performed transitions to two sources: a 'meritocratic' source represented by 'academic performance', and any further factors which cannot be justified from a meritocratic view.[7]

6. Model specification and observed correlations. Model 5 assumes that $\dot{C}$, representing the choice situation, solely depends on 'academic performance' ($\dot{Z}$). It is well possible, of course, that observations suggest that $\dot{C}$ depends on $\ddot{X}$ in ways not mediated through $\dot{Z}$ (see Ditton et al. (2005) for some empirical evidence). One might then consider the following modified model:

Model 6

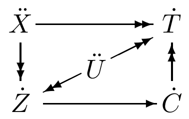
$$\begin{tikzcd} \ddot{X} \arrow[r, two heads] \arrow[d, two heads] \arrow[dr, two heads] & \dot{T} \\ \dot{Z} \arrow[r, two heads] & \dot{C} \arrow[u, two heads] \end{tikzcd}$$

There is then no longer a deterministic relationship between $\dot{C}$ and $\dot{Z}$; and the 'primary effect' of $\ddot{X}$ (as defined at the beginning of this section) cannot be interpreted solely in terms of 'academic performance'. Nevertheless, it is still possible to separate a primary and a secondary effect as suggested in § 3.2.

But now assume that observations suggest a correlation between $\dot{Z}$ and $\dot{T}$ even conditional on $\dot{C} = 1$. There are then different possibilities to modify Model 5, corresponding to different theoretical ideas. One possibility is to add an arrow from $\dot{Z}$ to $\dot{T}$. This would reflect the hypothesis that the degree of 'academic performance' not only determines the possibility of a choice, but in addition influences the final decision. A quite different possibility is to suppose an unobserved variable, say $\ddot{U}$, which, like $\ddot{X}$ influences both $\dot{Z}$ and $\dot{T}$. This would suggest the following model:

[7]Sometimes this interest in meritocratic evaluations is explicitly mentioned, see e.g. Schindler and Reimer (2010: 624).

Model 7



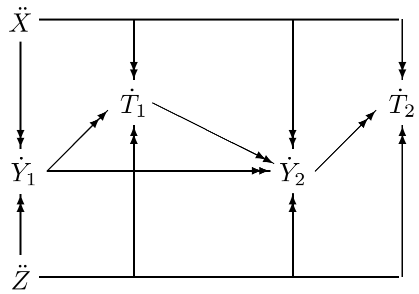
The example demonstrates that it is insufficient to think in terms of correlation (observed or theoretically postulated). There are always several different dependency relations between variables that can generate the observed correlations.

4. Educational outcomes and transitions

I now consider an enlarged version of the transition model that was discussed in Section 2 which includes effects of the educational outcome of a foregoing learning frame.

1. Including educational outcomes. I consider the following model which is a combination of the models considered in sections 1 and 2:

Model 8



As before, $\dot{Y}_j$ is the educational outcome, and $\dot{T}_j$ represents the transition. The model entails the following constraint: $\dot{Y}_2$ and $\dot{T}_2$ are only defined if $\dot{T}_1 = 1$. This must be taken into account when defining conditional effects on $\dot{Y}_2$ and $\dot{T}_2$.

2. Specification of dependency relations. Remembering the discussion of primary and secondary effects in Section 3, a critical question concerns how to specify the relationship between the educational outcome of a learning frame, $\dot{Y}_j$, and the subsequent transition, $\dot{T}_j$. Here I assume that a minimal level, say λ_j, is a necessary condition for making a transition:

$$\dot{T}_j = 0 \;\text{ if }\; \dot{Y}_j < \lambda_j \tag{31}$$

and that the probability of $\dot{T}_j\!=\!1$ depends in some way on the difference, $\dot{Y}_j - \lambda_j$. To illustrate a possible specification, I use

$$\Pr(\dot{T}_j\!=\!1|\ddot{X}\!=\!x, \ddot{Z}\!=\!z, \dot{Y}_j\!=\!y_j) = F_j(\alpha_j + x\beta_j + z\gamma_j + \eta_j(y_j - \lambda_j))\, I[y_j \geq \lambda_j] \tag{32}$$

where F_j is a link function (e.g., a normal or logistic distribution function).

A further question concerns the specification of how educational outcomes depend on previous conditions. In contrast to the models considered in Section 1, it is no longer sufficient to focus only on the expectation of $\dot{Y}_j$; one needs at least a specification of how the probability of $\dot{Y}_j \geq \lambda_j$ depends on explanatory variables.

3. Primary and secondary effects. Model 8 can be used to consider the distinction between primary and secondary effects of SES that was introduced in Section 3. Primary effects concern the effect of $\ddot{X}$ on the opportunity to make a transition. For the first transition, this effect can be defined by

$$\begin{aligned}&\Delta^s(I[\dot{Y}_1 \geq \lambda_1]; \ddot{X}[x', x''], \ddot{Z}=z) := \\ &\qquad \Pr(\dot{Y}_1 \geq \lambda_1 |\, \ddot{X}=x'', \ddot{Z}=z) - \Pr(\dot{Y}_1 \geq \lambda_1 |\, \ddot{X}=x', \ddot{Z}=z)\end{aligned} \tag{33}$$

Secondary effects, on the other hand, concern the effect of $\ddot{X}$ on actually making the transition, given that there is an opportunity. For the first transition, this effect can be defined by

$$\begin{aligned}&\Delta^s_c(\dot{T}_1; \ddot{X}[x', x''], \ddot{Z}=z, \dot{Y}_1 \geq \lambda_1) := \\ &\qquad \Pr(\dot{T}_1=1 |\, \ddot{X}=x'', \ddot{Z}=z, \dot{Y}_1 \geq \lambda_1) - \Pr(\dot{T}_1=1 |\, \ddot{X}=x', \ddot{Z}=z, \dot{Y}_1 \geq \lambda_1)\end{aligned} \tag{34}$$

The secondary effect is conditional on $\dot{Y}_1 \geq \lambda_1$ and must be distinguished from the unconditional effect

$$\Delta^s_u(\dot{T}_1; \ddot{X}[x', x''], \ddot{Z}=z) := \Pr(\dot{T}_1=1 |\, \ddot{X}=x'', \ddot{Z}=z) - \Pr(\dot{T}_1=1 |\, \ddot{X}=x', \ddot{Z}=z) \tag{35}$$

This is the effect that would be estimated when using the sequential transition model that was considered in Section 2 and which does not allow distinguishing between primary and secondary effects.

To illustrate, I assume that the relationship between $\dot{Y}_1$ and $\dot{T}_1$ is given by (32), and $\dot{Y}_1$ is normally distributed with mean

$$\mathrm{E}(\dot{Y}_1 | \ddot{X}=x, \ddot{Z}=z) = \alpha^y_1 + x\beta^y_1 + z\gamma^y_1 \tag{36}$$

and a unit variance. The primary effect can then be calculated from

$$\Pr(\dot{Y}_1 \geq \lambda_1 |\, \ddot{X}=x, \ddot{Z}=z) = \Phi(\alpha^y_1 + x\beta^y_1 + z\gamma^y_1 - \lambda_1) \tag{37}$$

(where Φ denotes the standard normal distribution function), the unconditional effect can be calculated from

$$\begin{aligned}&\Pr(\dot{T}_1=1 |\, \ddot{X}=x, \ddot{Z}=z) = \\ &\quad \int_{y \geq \lambda_1} \Pr(\dot{T}_1=1 |\, \ddot{X}=x, \ddot{Z}=z, \dot{Y}_1=y)\, f(y|x,z)\, dy = \\ &\quad \int_{y \geq \lambda_1} F_1(\alpha_j + x\beta_j + z\gamma_j + \eta_j(y - \lambda_j))\, \phi(y - (\alpha^y_1 + x\beta^y_1 + z\gamma^y_1))\, dy\end{aligned}$$

(where ϕ denotes the standard normal density function), and the secondary effect can be calculated from

$$\Pr(\dot{T}_1=1 |\, \ddot{X}=x'', \ddot{Z}=z, \dot{Y}_1 \geq \lambda_1) = \frac{\Pr(\dot{T}_1=1 |\, \ddot{X}=x, \ddot{Z}=z)}{\Pr(\dot{Y}_1 \geq \lambda_1 |\, \ddot{X}=x, \ddot{Z}=z)}$$

For a numerical illustration, I use a logit specification for F_1 and assume $\alpha_1 = \beta_1 = \gamma_1 = \eta_1 = 1$, and $\alpha^y_1 = 0.5$, $\beta^y_1 = \gamma^Y_1 = 0.7$, $\lambda_1 = 1$. For $z=0$, one finds:

x	$\Pr(\dot{Y}_1 \geq \lambda_1 \mid x, z)$	$\Pr(\dot{T}_1=1 \mid x, z, \dot{Y}_1 \geq \lambda_1)$	$\Pr(\dot{T}_1=1 \mid x, z)$
−1	0.115	0.617	0.071
0	0.309	0.825	0.255
1	0.579	0.940	0.544
2	0.816	0.982	0.801

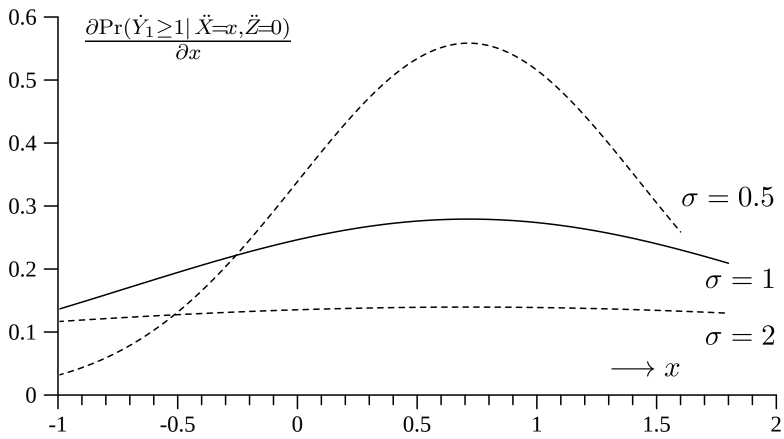


Figure 3 Dependence of the marginal effect (38), with $\ddot{Z} = 0$, on values of σ. Parameters: $\alpha_1^y = 0.5$, $\beta_1^y = \gamma_1^y = 0.7$, $\lambda_1 = 1$.

Effects depend on x, for example:

$$\begin{aligned}
&\Delta^s(I[\dot{Y}_1 \geq \lambda_1]; \ddot{X}[0,1], \ddot{Z}=0) &&= 0.579 - 0.309 = 0.270 \\
&\Delta^s_c(\dot{T}_1; \ddot{X}[0,1], \ddot{Z}=0, \dot{Y}_1 \geq 1) &&= 0.940 - 0.825 = 0.115 \\
&\Delta^s_u(\dot{T}_1; \ddot{X}[0,1], \ddot{Z}=0) &&= 0.544 - 0.255 = 0.289
\end{aligned}$$

4. Educational outcomes and primary effects. Given the understanding that primary effects concern opportunities for transitions, they must be distinguished from effects on educational outcomes. This can be illustrated with the specifications assumed in the previous paragraph. Using (36) for the educational outcome, $\dot{Y}_1$, the marginal effect of $\ddot{X}$ is β_1^y. In contrast, using (37), the marginal effect of $\ddot{X}$ on $I[\dot{Y}_1 \geq \lambda_1]$ is

$$\frac{\partial \Pr(\dot{Y}_1 \geq \lambda_1 | \ddot{X}=x, \ddot{Z}=z)}{\partial x} = \phi(\alpha_1^y + x\beta_1^y + z\gamma_1^y - \lambda_1)\, \beta_1^y \tag{38}$$

This shows that the primary effect is not only determined by β_1^y, but also by values of explanatory variables and, most important, by λ_1.

This also entails that the primary effect depends on the degree of inequality in the educational outcomes. This can be illustrated by introducing a parameter for the variance of $\dot{Y}_1$:

$$\dot{Y}_1 = \alpha_1^y + x\beta_1^y + z\gamma_1^y + \sigma \dot{U}_1 \tag{39}$$

where $\dot{U}_1$ is a normally distributed random variable with unit variance. The derivation of (37) was based on $\sigma = 1$. Starting from (39), one gets

$$\Pr(\dot{Y}_1 \geq \lambda_1 | \ddot{X}=x, \ddot{Z}=z) = \Phi\big(\frac{1}{\sigma}[\alpha_1^y + x\beta_1^y + z\gamma_1^y - \lambda_1]\big) \tag{40}$$

Consequences of an increasing variance of the educational outcomes depend on the expectation of $\dot{Y}_1$:

if $\mathrm{E}(\dot{Y}_1|x,z) > \lambda_1$, the probability of $\dot{Y}_1 \geq \lambda_1$ decreases, and
if $\mathrm{E}(\dot{Y}_1|x,z) < \lambda_1$, the probability of $\dot{Y}_1 \geq \lambda_1$ increases.

Also the marginal effects of the explanatory variables depend on the variance of the educational outcomes. Figure 3 illustrates how the marginal effect of $\ddot{X}$, as defined in (38), depends on values of σ. Notice that these effects also depend on values of $\ddot{Z}$.

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